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Large Scale Machine Learning: Decision Trees

CS246: Mining Massive Datasets
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http://cs246.stanford.edu



New Topic: ML!

High dim.

Locality sensitive hashing

Clustering

Dimensional ity reduction

Graph data

PageRank, SimRank

Community Detection

Spam Detection

Infinite data

Filtering data streams

Web advertising

Queries on streams

Machine learning

Decision Trees

Random Forest, GBDT

Neural Networks, GNNs **Apps**

Recommen der systems

Association Rules

Duplicate document detection

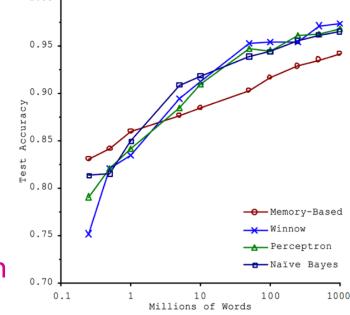
Why Large-Scale ML?

Brawn or Brains?

In 2001, Microsoft researchers ran a test to evaluate
 4 different approaches to ML-based language
 translation

Findings:

- Size of the dataset used to train the model mattered more than the model itself
- As the dataset grew large, performance difference between the models became small



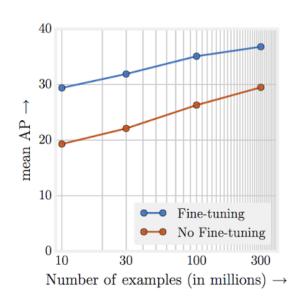
Why Large-Scale ML?

The Unreasonable Effectiveness of Data

 In 2017, Google revisited the same type of experiment with the latest Deep Learning models in computer vision

Findings:

- Performance increases logarithmically based on volume of training data
- Complexity of modern ML models (i.e., deep neural nets) allows for even further performance gains



Large datasets + large ML models => amazing results!!

Why Worry About Non-Deep Models?

A few reasons why this is important:

- They outperform DL models in certain taks.
- Deep models are often hard to scale and require lots of data. Traditional models allow you to encode prior knowledge better and give you more control.
- Combine: ideas from several ML models, e.g., GNNs
- Rule of thumb: If working on a well understood problem use deep learning. If working on a new problem use techniques we'll discuss here.

Decision Trees, Random Forests, AdaBoost and GBDTs

Preface: Decision Trees

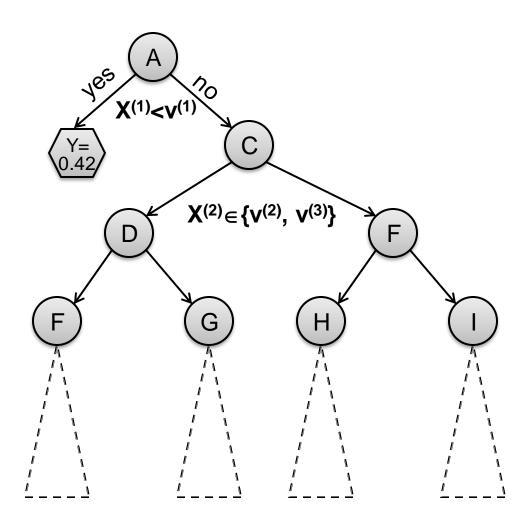
- Decision trees are part of ML since 1980s
 - Introduced by Leo Breiman in 1984
 - Notable algorithms: ID3, C4.5
- More recent innovations include:
 - Boosted decision trees (gradient boosted DT)
 - Random forest
- Even though DTs are old, hand-engineered and heuristic, they are a method of choice for tabular data and for Kaggle competitions.

Decision Tree Learning

- Given one attribute (e.g., lifespan), try to predict the value of new people's lifespans by a subset of the other available attributes
- Input attributes:
 - **d** features/attributes: $x^{(1)}$, $x^{(2)}$, ... $x^{(d)}$
 - Each $x^{(j)}$ has **domain** O_j
 - Categorical: $O_i = \{male, female, nonbinary\}$
 - Numerical: $H_i = (1, 200)$
 - Y is output variable with domain O_Y :
 - Categorical: Classification e.g. Y = eye color
 - Numerical: Regression e.g. Y = lifespan
- Data D:
 - n examples (x_i, y_i) where x_i is a d-dim feature vector, $y_i \in O_Y$ is output variable
- Task:
 - lacksquare Given an input data vector $oldsymbol{x}$ predict output label $oldsymbol{y}$

Decision Trees

 A Decision Tree is a tree-structured plan of a set of attributes to test in order to predict the output



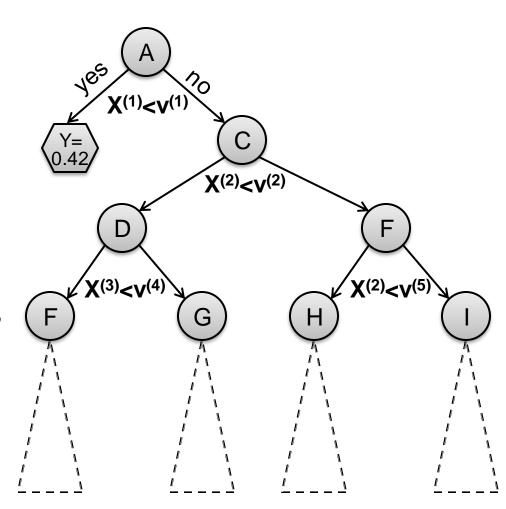
Decision Trees

Decision trees:

- Split the data at each internal node
- Each leaf node makes a prediction

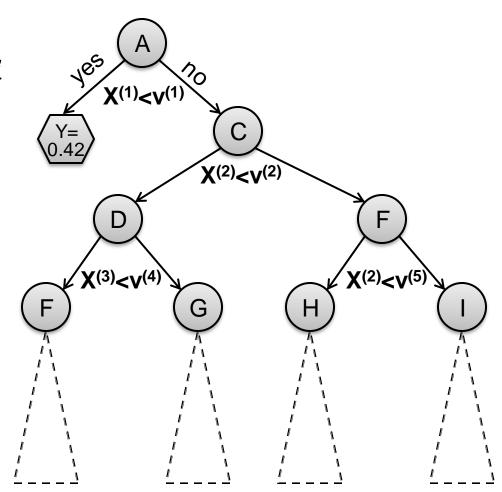
Lecture today:

- Binary splits: $X^{(j)} < v$
- Numerical attributes
- Regression

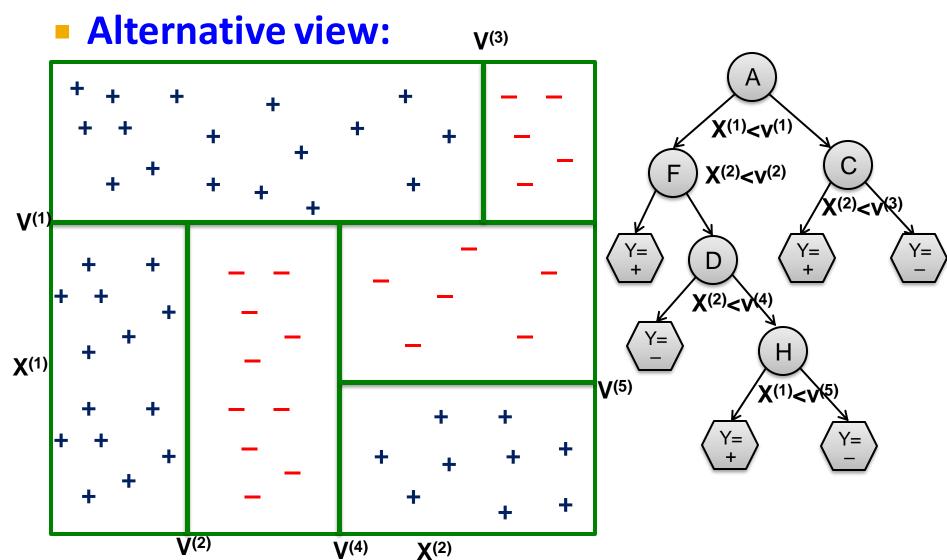


How to make predictions?

- Input: Example x_i
- Output: Predicted \hat{y}_i
- "Drop" x_i down the tree until it hits a leaf node
- Predict the value stored in the leaf that x_i hits

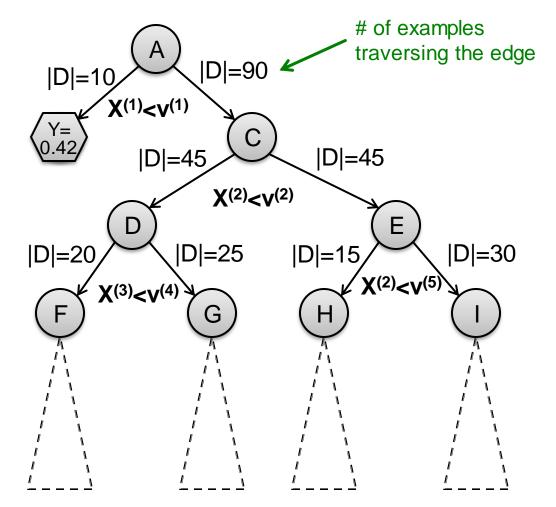


Decision Trees: feature space



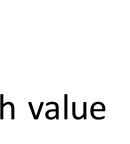
How to construct a tree?

• Training dataset D^* , $|D^*| = 100$ examples

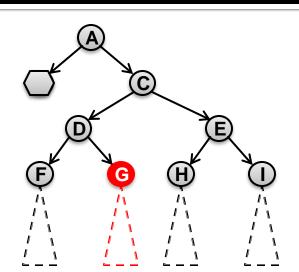


How to construct a tree?

- Imagine we are currently at some node G
 - Let D_G be the data that reaches G
- There is a decision we have to make: Do we continue building the tree?



- If yes, which variable and which value do we use for a split?
 - Continue building the tree recursively
- If not, how do we make a prediction?
 - We need to build a "predictor node"



3 steps in constructing a tree

```
Algorithm 1 | BuildSubtree
Require: Node n, Data D \subseteq D^*
 1: (n \to \text{split}, D_L, D_R) = \text{FindBestSplit}(D)
 2: if StoppingCriteria(D_L) then
 3: n \rightarrow \text{left\_prediction} = \text{FindPrediction}(D_L)(3)
 4: else
 5:
                   BuildSubtree (n \rightarrow \text{left}, D_L)
 6: if StoppingCriteria(D_R) then
        n \rightarrow \text{right\_prediction} = \text{FindPrediction}(D_R)
 8: else
                   BuildSubtree (n \rightarrow \text{right}, D_R)
 9:
```

Requires at least a single pass over the data!

How to construct a tree?

(1) How to split? Pick attribute & value that optimizes some criterion

- Regression: Purity
 - Find split (X⁽ⁱ⁾, v) that creates D, D_L, D_R: parent, left, right child datasets and maximizes:

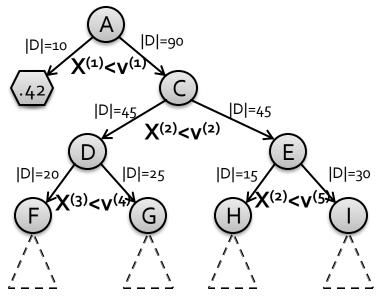
$$|D|=10 \qquad X^{(1)} < V^{(1)} \qquad C \qquad |D|=45 \qquad X^{(2)} < V^{(2)} \qquad E \qquad |D|=30 \qquad |D|=25 \qquad |D|=15 \qquad |D|=30 \qquad |D|=30$$

$$|D| \cdot Var(D) - (|D_L| \cdot Var(D_L) + |D_R| \cdot Var(D_R))$$

$$extbf{Var}(D) = rac{1}{|D|} \sum_{i \in D} (y_i - \overline{y})^2$$
 ... variance of y_i in D

How to construct a tree?

- (1) How to split? Pick attribute & value that optimizes some criterion
- Classification: Information Gain
 - Measures how much a given attribute X tells us about the class Y
 - IG(Y | X): We must transmit Y over a binary link. How many bits on average would it save us if both ends of the line knew X?

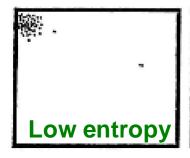


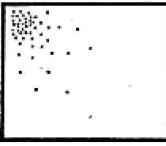
Why Information Gain? Entropy

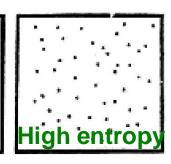
The entropy of *X*:

$$H(X) = -\sum_{j=1}^{m} p(X_j) \log p(X_j)$$

- "High Entropy": X is from a uniform (boring) distribution
 - A histogram of the frequency distribution of values of X is flat
- "Low Entropy": X is from a varied (peaks/valleys) distrib.
 - A histogram of the frequency distribution of values of X would have many lows and one or two highs







Why Information Gain? Entropy

Suppose I want to predict Y and I have input X

- X = College Major
- Y = Likes "Casablanca"

X	Y	
Math	Yes	
History	No	
CS	Yes	
Math	No	
Math	No	
CS	Yes	
Math	Yes	
History	No	

From this data we estimate

•
$$P(Y = Yes) = 0.5$$

$$P(X = Math \& Y = No) = 0.25$$

$$P(X = Math) = 0.5$$

$$P(Y = Yes | X = History) = 0$$

Note:

•
$$H(Y) = -\frac{1}{2}\log_2(\frac{1}{2}) - \frac{1}{2}\log_2(\frac{1}{2}) = \mathbf{1}$$

$$- H(X) = 1.5$$

Why Information Gain? Entropy

- Suppose I want to predict Y and I have input X
 - X = College Major
 - Y = Likes "Casablanca"

X	Y	
Math	Yes	
History	No	
CS	Yes	
Math	No	
Math	No	
CS	Yes	
Math	Yes	
History	No	

Def: Specific Conditional Entropy

• H(Y | X = v) = The entropy of Y among only those records in which X has value v

Example:

- H(Y|X = Math) = 1
- $\blacksquare H(Y|X = History) = 0$
- -H(Y|X=CS)=0

Why Information Gain?

- Suppose I want to predict Y and I have input X
 - X = College Major
 - Y = Likes "Casablanca"

X	Υ	
Math	Yes	
History	No	
CS	Yes	
Math	No	
Math	No	
CS	Yes	
Math	Yes	
History	No	

Def: Conditional Entropy

- $H(Y \mid X)$ = The average specific conditional entropy of Y
 - = the entropy of Y, conditioned X, if you choose a record at random

$$= \sum_{j} P(X = v_j) H(Y|X = v_j)$$

Why Information Gain?

Suppose I want to predict Y and I have input X

• H(Y | X) = The average specific conditional entropy of Y

X	Y	
Math	Yes	
History	No	
CS	Yes	
Math	No	
Math	No	
CS	Yes	
Math	Yes	

No

$$= \sum_{j} P(X = v_j) H(Y|X = v_j)$$

Example:

$V_{\rm j}$	P(X=v _j)	$H(Y X=v_j)$
Math	0.5	1
History	0.25	0
CS	0.25	0

So: H(Y|X)=0.5*1+0.25*0+0.25*0 =**0.5**

History

Why Information Gain?

Suppose I want to predict Y and I have input X

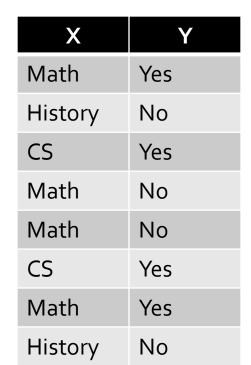


• IG(Y|X) = I must predict Y. How much information do I get about Y if I knew X?

$$IG(Y|X) = H(Y) - H(Y|X)$$



- H(Y) = 1
- H(Y|X) = 0.5
- Thus IG(Y|X) = 1 0.5 = 0.5



What is Information Gain used for?

- Suppose you are trying to predict whether someone is going to live past 80 years
- From historical data you might find:
 - IG(LongLife | HairColor) = 0.01
 - IG(LongLife | Smoker) = 0.4
 - IG(LongLife | Gender) = 0.25
 - IG(LongLife | LastDigitOfSSN) = 0.00001
- IG tells us how much information about Y is contained in X
 - So attribute X that has high IG(Y|X) is a good split!

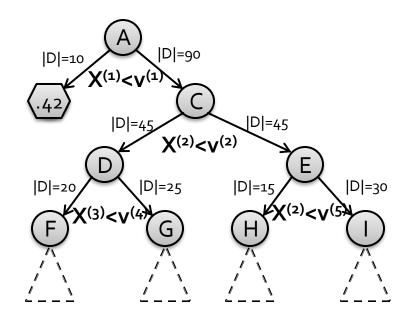
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```

When to stop?

(2) When to stop?

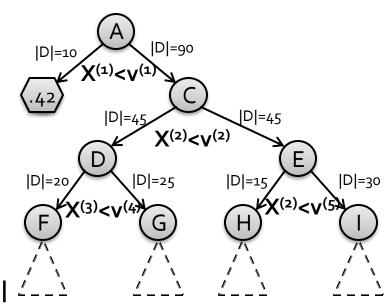
- Many different heuristic options
- Two ideas:
 - (1) When the leaf is "pure"
 - The target variable does not vary too much: $Var(y) < \varepsilon$
 - (2) When # of examples in the leaf is too small
 - For example, $|D| \le 100$



How to predict?

(3) How to predict?

- Many options
 - Regression:
 - Predict average y_i of the examples in the leaf
 - Build a linear regression model on the examples in the leaf
 - Classification:
 - Predict most common y_i of the examples in the leaf



Decision Trees

Characteristics

- Classification & Regression
 - Multiple (~10) classes
- Real valued and categorical features
- Few (hundreds) of features
- Usually dense features
- Complicated decision boundaries
 - Early stopping to avoid overfitting!
- Example applications
 - User profile classification
 - Landing page bounce prediction

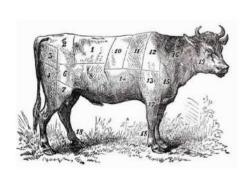
Decision Trees

- Decision trees are the single most popular data mining tool:
 - Interpretable
 - Easy to implement
 - Easy to use
 - Computationally cheap
 - It's possible to mitigate overfitting (i.e., with ensemble methods)
 - They do classification as well as regression!

Decision Trees: Learning Ensembles

Learning Ensembles

- Learn multiple trees and combine their predictions
 - Fix overfitting/underfitting problem in decision trees
 - Gives better performance in practice
 - The "wisdom of the crowds"
- The parable of the ox (Sir Francis Galton, 1906)
 - 787 people guessed the weight of an ox
 - Avg crowd guess: 1,197 pounds
 - True weight: 1,198 pounds



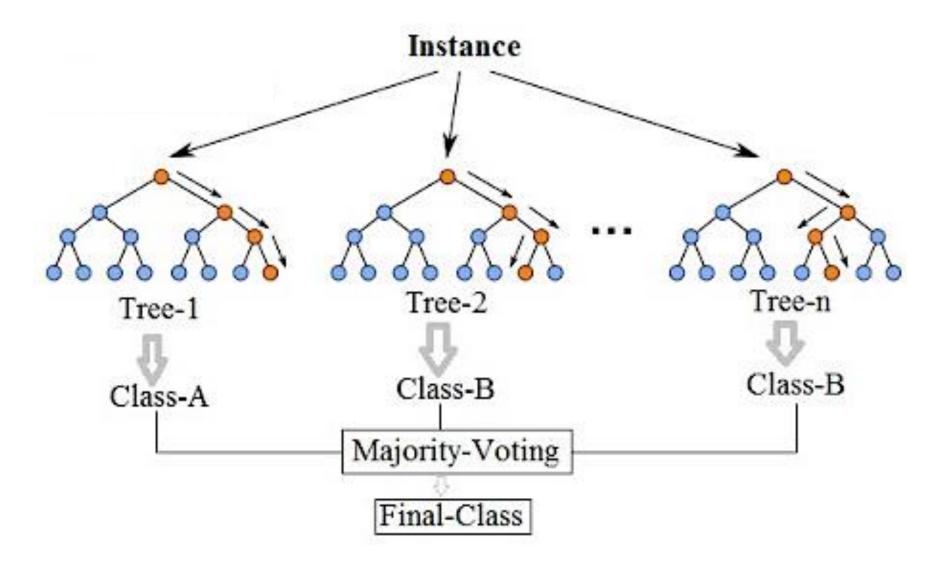


Learning Ensembles

Bagging (bootstrap aggregation):

- Learns multiple trees <u>in parallel</u> over independent samples of the training data
- 1) Bootstrapping: Given a dataset D on n data points: Create multiple datasets D' of n points by sampling from D with replacement:
 - 33% points in D' will be duplicates, 66% will be unique
 - 2) Parallel training: Train decision
 trees on samples independently and in parallel
 - 3) Aggregation: Depending on the task, an average or majority of the predictions are computed for a more accurate estimate

(1): Bagging Decision Trees



(1): Instance Bagging

Decision trees are greedy

- They choose which variable to split on using a greedy algorithm that maximizes purity or information gain
- Even with Bagging, the decision trees can have a lot of structural similarities and correlation in their predictions
 - If one feature is very strong predictor, then every tree will select it, causing trees to be correlated.
- But ensemble learning works best with independent predictors

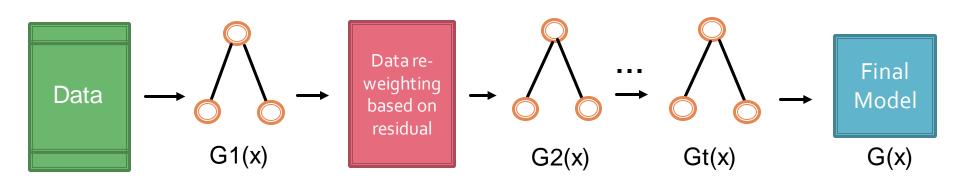
(2) Improvement: Random Forests

- Train a Bagged Decision Tree
- But use a modified tree learning algorithm that selects (at each candidate split) a random subset of features
 - If we have d features, consider \sqrt{d} random features
- This is called: <u>Feature bagging</u>
 - Benefit: Breaks correlation between trees
- Random Forests achieve state-of-the-art results in many classification problems!

(3): Boosting

Boosting: Another ensemble learning algorithm

- Combines the outputs of many "weak" classifiers to produce a powerful "committee"
- Learns multiple trees sequentially, each trying to improve upon its predecessor
- Final classifier is weighted sum of the individual classifiers



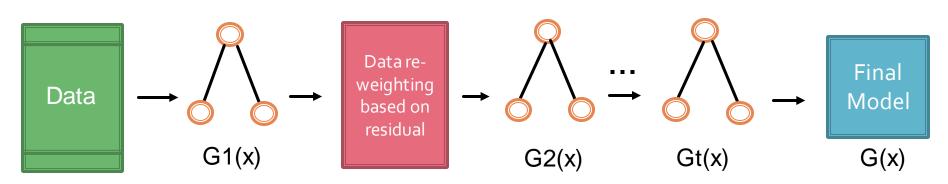
Residual: Difference between prediction and ground truth

Χ

(3): Boosting

We will show 2 examples:

- Example 1: AdaBoost
 - Where each $G_t(x)$ is a one-level decision tree
- Example 2: Gradient Boosted Decision Trees
 - Where each $G_t(x)$ is a multi-level decision tree



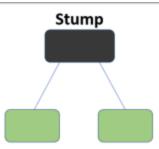
X

Residual: Difference between prediction and ground truth

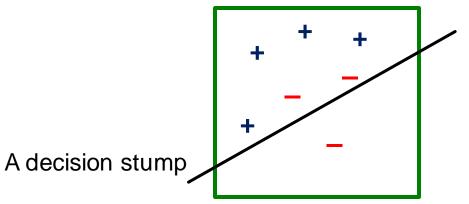
AdaBoost: Weak learner

Decision "stumps":

1-level decision tree



- A decision boundary based on one feature
 - E.g.: If someone is not a smoker, then predict them to live past 80 years old
- Building blocks of AdaBoost algorithm
- Decision stump is a weak learner



Boosting theory: if weak learners have >50% accuracy then we can learn a perfect classifier.

Build Decision Trees with AdaBoost

Suppose we have training data $\{(x_i, y_i)\}_{i=1}^N$, $y_i \in \{1, -1\}$

- Initialize equal weights for all observations $w_i = 1/N$
- At each iteration t:
 - 1. Train a stump G_t using data weighted by W_i
 - Compute the misclassification error adjusted by Wi
 - 3. Compute the weight of the current tree α_t
 - 4. Reweight each observation based on prediction accuracy

Update Step

Calculate the weighted misclassification error

$$err_t = \frac{\sum_{i=1}^{N} w_i I(y_i \neq G_t(x_i))}{\sum_{i=1}^{N} w_i}$$

Use the error score to weight the current tree in the final classifier:

$$\alpha_t = log\left(\frac{1 - err_t}{err_t}\right)$$

A classifier with 50% accuracy is given a weight of zero;

 Use misclassification error and tree weight to reweight the training data:

$$w_i \leftarrow w_i exp[\alpha_t I(y_i \neq G_t(x_i))]$$

Training instances that are harder to classify get higher weight

Final Prediction

Final prediction is a weighted sum of the predictions from each stump:

$$G(x) = sign\left[\sum_{t=1}^{T} \alpha_t G_t(x)\right]$$

 More accurate trees are weighted higher in the final model

AdaBoost: Summary

- 1. Initialize the observation weights $w_i = 1/N, i = 1, 2, ..., N$.
- 2. For m=1 to M:
 - (a) Fit a classifier $G_m(x)$ to the training data using weights w_i .
 - (b) Compute

(1) Train a stump

$$\operatorname{err}_m = \frac{\sum_{i=1}^N w_i I(y_i \neq G_m(x_i))}{\sum_{i=1}^N w_i}.$$
 (2) Compute error

- (c) Compute $\alpha_m = \log((1 \text{err}_m)/\text{err}_m)$. (3) Compute tree weight
- (d) Set $w_i \leftarrow w_i \cdot \exp[\alpha_m \cdot I(y_i \neq G_m(x_i))], i = 1, 2, \dots, N.$
- 3. Output $G(x) = \operatorname{sign} \left[\sum_{m=1}^{M} \alpha_m G_m(x) \right]$.

(4) Reweight data

AdaBoost Conclusion

- Iteratively train weak learners (decision stumps) to form a strong model:
 - Trees with high accuracy are given more weights in the final model
 - Misclassified data get higher weights in the next iteration
- AdaBoost is the equivalent to additive training with the exponential loss (Friedman et al. 2000)
- We will talk about additive training in more general scenarios next!

Gradient Boosted Decision Trees

Gradient Boosted Decision Trees

- Idea: Optimize an Additive model
 - Additive prediction model:

$$\hat{y}_i = \sum_{t=1}^{T} f_t \left(x_i \right)$$

- Here f_t can be multi-level!
- Objective (cost) function:

obj
$$(\theta) = \sum_{i=1}^{N} l(y_i, \hat{y}_i) + \sum_{t=1}^{T} \omega(f_t)$$

• $\omega(f_t)$ is a regularization term that models the complexity of the tree.

Gradient Boosted Decision Trees

- Use Additive model to train sequentially:
 - Start from constant prediction, add a new decision tree f_i each time:

$$\hat{y}_i^{(0)}=0$$
 $\hat{y}_i^{(1)}=f_1(x_i)=\hat{y}_i^{(0)}+f_1(x_i)$ $\hat{y}_i^{(2)}=f_1(x_i)+f_2(x_i)=\hat{y}_i^{(1)}+f_2(x_i)$ \cdots $\hat{y}_i^{(t)}=\sum_{k=1}^t f_k(x_i)=\hat{y}_i^{(t-1)}+f_t(x_i)$ Prediction at Keep predictions from previous rounds

How to decide which f to add?

- Prediction at round t is: $\hat{y}_i^{(t)} = \hat{y}_i^{(t-1)} + f_t(x_i)$
 - Where we need to decide what $f_t()$ to add
- Goal: Find tree $f_t(\cdot)$ that minimizes loss $l(\cdot)$:

$$obj^{(t)} = \sum_{i=1}^{n} l\left(y_i, \hat{y}_i^{(t)}\right) + \omega\left(f_t\right)$$

- y_i : The ground-truth label
- $\widehat{y_i}^{(t-1)} + f_t(x_i)$: The prediction made at round t
- $\omega(f_t)$: The model complexity

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- y_i : The ground-truth label
- $\widehat{y_i}^{(t-1)} + f_t(x_i)$: The prediction made at round t
- $\omega(f_t)$: The model complexity

How to decide which f to add?

$$obj^{(t)} = \sum_{i=1}^{n} l \left(y_i, \hat{y}_i^{(t-1)} + f_t(x_i) \right) + \omega(f_t)$$

Take Taylor expansion of the objective:

$$g(x + \Delta) \approx g(x) + g'(x)\Delta + \frac{1}{2}g''(x)\Delta^{2}$$

So, we get the approximate objective:

$$ext{obj}^{(t)} = \sum_{i=1}^n [\underline{l(y_i, \hat{y}_i^{(t-1)})} + g_i f_t(x_i) + rac{1}{2} h_i f_t^2(x_i)] + \omega(f_t)$$

We can ignore this part, since we are optimizing over f_t

where:

$$g_i = \partial_{\hat{y}^{(t-1)}} l(y_i, \hat{y}^{(t-1)}), \quad h_i = \partial_{\hat{y}^{(t-1)}}^2 l(y_i, \hat{y}^{(t-1)})$$

Our New Goal

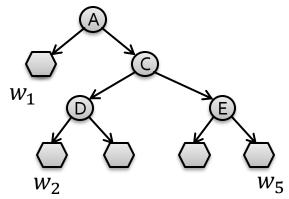
• Our new goal: Find tree f_t that:

$$\sum_{i=1}^n [g_i f_t(x_i) + rac{1}{2} h_i f_t^2(x_i)] + \omega(f_t)$$

- Why spend so much efforts to derive the objective, why not just grow trees ...
 - Theoretical benefit: Know what we are learning
 - Engineering benefit:
 - ullet g and h comes from definition of loss function
 - Learning f_t only depends on the objective via g and h
 - We can now directly learn trees that optimize the loss (rather than using some heuristic procedure)

Define a Tree

- Every leaf j have a weight w_j
 - We will predict w_i for any data belongs to leaf j



$$f_t(x) = w_{\underline{q(x)}}$$

q(x) indicate the leaf node that data point x belongs to

Define complexity of tree f as:

$$\Omega(f) = \gamma * T + \frac{1}{2}\lambda \sum_{i}^{I} w_{j}^{2}$$

T... number of leaves of tree f

 γ ... cost adding a leaf to the tree f

Revisiting the Objective

- Define:
- The set of examples in the leaf j:

$$I_j = \{i | q(x_i) = j\}$$
 q(x) denotes the leaf that data point x belongs to

The parameters that depend on the loss:

$$G_j = \sum_{i \in I_j} g_i$$
 $H_j = \sum_{i \in I_j} h_i$

Then the objective function becomes:

$$\mathrm{obj}^{(t)} = \sum_{j=1}^T [G_j w_j + rac{1}{2} (H_j + \lambda) w_j^2] + \gamma T$$

How to find a single tree f_t

Given a tree f_t , we know how to

Calculate the score for f:

$$Obj = -\frac{1}{2} \sum_{j=1}^{T} \frac{G_j^2}{H_j + \lambda} + \gamma T$$

And then set optimal weights for the chosen f:

$$w_j^* = -\frac{G_j}{H_j + \lambda}$$

In principle we could:

• Enumerate possible tree structures f and take the one that minimizes Obj

How to find a single tree f_t

- In practice we grow tree greedily:
 - Start with tree with depth 0
 - For each leaf node in the tree, try to add a split
 - The change of the objective after adding a split is:

$$Gain = \frac{1}{2} [\frac{G_L^2}{H_L + \lambda} + \frac{G_R^2}{H_R + \lambda} - \frac{(G_L + G_R)^2}{H_L + H_R + \lambda}] - \gamma$$
 Score of Score of left child right child

- Take the split that gives best gain
- Next: How to find the best split?

How to Find the Best Split?

For each node, enumerate over all features

- For each feature, sort the instances by feature value
- Use a linear scan to decide the best split along that feature
- Take the best split solution along all the features

Pre-stopping:

- Stop split if the best split have negative gain
- But maybe a split can benefit future splits.

Post-Prunning:

 Grow a tree to maximum depth, recursively prune all the leaf splits with negative gain.

Summary: GBDT Algorithm

- Add a new tree $f_t(x)$ in each iteration
 - Compute necessary statistics for our objective

$$g_i = \partial_{\hat{y}^{(t-1)}} l(y_i, \hat{y}^{(t-1)}), \quad h_i = \partial_{\hat{y}^{(t-1)}}^2 l(y_i, \hat{y}^{(t-1)})$$

• Greedily grow the tree that minimizes the objective:

$$Obj = -\frac{1}{2} \sum_{j=1}^{T} \frac{G_j^2}{H_j + \lambda} + \gamma T$$

- Add $f_t(x)$ to our ensemble model

$$y^{(t)} = y^{(t-1)} + \epsilon f_t(x_i)$$
 usually set around 0.1

 ϵ is called step-size or shrinkage, usually set around 0.1

Goal: prevent overfitting

Repeat until we user M ensemble of trees

XGBoost

- XGBoost: eXtreme Gradient Boosting
 - A highly scalable implementation of gradient boosted decision trees with regularization

Widely used by data scientists and provides state-of-theart results on many problems!

- System optimizations:
 - Parallel tree constructions using column block structure
 - Distributed Computing for training very large models using a cluster of machines.
 - Out-of-Core Computing for very large datasets that don't fit into memory.

Summary of the Lecture

- Basics of supervised learning
- Decision Trees
 - Key idea: split data at each internal node, make prediction at each leaf node
 - How to construct a tree: Information Gain
- Ensemble of decision trees:
 - Bagging: Random forests
 - Boosting: Boosted decision trees